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The contact problem for a strip cover plate interacting with an elastic half-space☆

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ABSTRACT

The problem of the contact interaction of a cover plate having a base in the form of a narrow rectangle, not resistant to bending deformations but stable to stretching, with an elastic isotropic half-space, loaded at infinity by a stretching force, directed parallel to the boundary of the half-space, is considered. The problem is reduced to an integral equation of the first kind and an approximate method of solving it is indicated. Formulae for the contact shear stress are obtained.

1. Formulation of the problem

Investigation of the integral equation. We will consider, in a rectangular system of coordinates x, y, z, the elastic half-space $z \le 0$, loaded at infinity with a tension p. The boundary $z=0$ is strengthened with a punch having the form of a narrow rectangle of length $2a$ and width 2ε $(a/\varepsilon \gg 1)$ in plan.

The boundary conditions for the elastic half-space have the form

$$\sigma_{zz}(x, y, 0) = 0, \quad \sigma_{zy}(x, y, 0) = 0$$
$$\tau_{zx}(x, y, 0) = \tau(x, y) \quad (|x| \le a, |y| \le \varepsilon), \quad \tau_{zx}(x, y, 0) = 0 \ (|x| > a, |y| > \varepsilon)$$
$$u(x, y, 0) = 0 \ (|x| \le a, |y| \le \varepsilon) \tag{1.1}$$

At infinity

$$\sigma_{xx}(x, y, z) = p \tag{1.2}$$

and the remaining stresses vanish. Here σ_{xx}, σ_{zz}, σ_{zy}, τ_{zx} are the stresses in the half-space, $\tau(x, y)$ is the contact shear between the lower surface of the cover plate and the boundary of the half-space, and u is the displacement of the points of the half-space along the x axis.

Using the formula for the displacement,[1] we reduce problem (1.1), (1.2) to determining the function $\tau(x, y)$ from the integral equation

$$\int_{-a}^{a} d\xi \int_{-\varepsilon}^{\varepsilon} \tau(\xi, \eta)\left(\frac{2(1-\nu)}{R} + \frac{2\nu(\xi - x)^2}{R^3}\right) d\eta = -\frac{4\pi p x}{1+\nu}; \quad R = \sqrt{(\xi - x)^2 + \eta^2} \tag{1.3}$$

Here ν is Poisson's ratio of the half-space.

We will use Galin's assumption[2] that the pressure distribution under the punch in a transverse direction will be the same as is obtained by solving the corresponding plane problem, i.e., we will seek the contact shear stress in the form[3]

$$\tau(x, y) = \frac{\tau(x)}{\sqrt{\varepsilon^2 - y^2}}, \quad |x| < a, \quad |y| < \varepsilon \tag{1.4}$$

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We substitute expression (2.1) into integral equation (1.3) and use the well-known formulae from Ref. 4 (3.152(4) and 3.158(5)) to represent the integrals in terms of complete elliptic integrals of the first kind $\mathbf{K}(k)$ and the second kind $\mathbf{E}(k)$.

After reduction, integral Eq. (1.3) takes the form

$$\int_{-a}^{a}\tau(\xi)K\left(\frac{\xi-x}{\varepsilon}\right)d\xi = -\frac{\pi p x\varepsilon}{1+\nu}$$
$$K(t) = \frac{1}{\sqrt{1+t^2}}\left((1-\nu)\mathbf{K}\left(\frac{1}{\sqrt{1+t^2}}\right)+\nu\mathbf{E}\left(\frac{1}{\sqrt{1+t^2}}\right)\right) \quad (1.5)$$

We separate the logarithmic singularity $-(1-\nu)\ln|t|$ in the kernel $K(t)$ and introduce the dimensionless quantities

$$x' = \frac{x}{a}, \quad \xi' = \frac{\xi}{a}, \quad \varepsilon' = \frac{\varepsilon}{a}, \quad \varphi(\xi') = \frac{\tau(\xi)}{pa}$$

Omitting the primes, we write Eq. (1.5) in the form

$$-\int_{-1}^{1}\varphi(\xi)\ln\frac{|\xi-x|}{\varepsilon}d\xi = -\frac{\pi x\varepsilon}{1-\nu^2}-\int_{-1}^{1}\varphi(\xi)F\left(\frac{\xi-x}{\varepsilon}\right)d\xi$$
$$F(t) = \ln|t| + K(t) \quad (1.6)$$

2. Numerical solution

Eq. (1.6) can be solved using a modified Multhopp-Kalandia method,[5] which is basically as follows. The solution of Eq. (1.6) can be represented in the form[6]

$$\varphi(x) = \Phi(x)/\sqrt{1-x^2} \quad (2.1)$$

where the function $\Phi(x)$ is at least continuous. Further, at the nodes

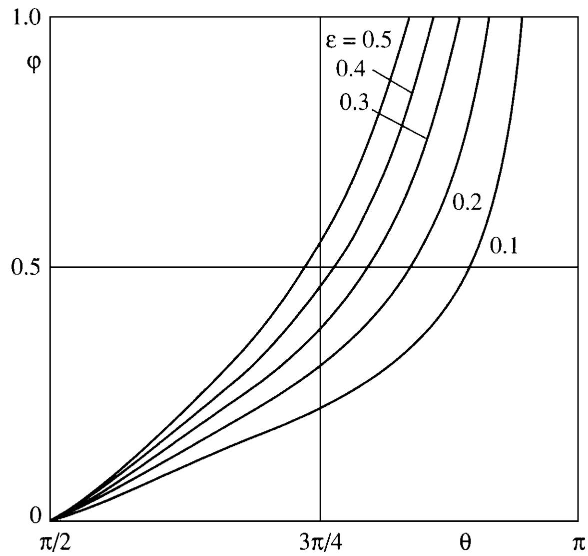
$$x_n = \cos\theta_n; \quad \theta_n = \frac{\pi(n-1/2)}{N}, \quad n = 1, 2, \ldots, N \quad (2.2)$$

a Lagrange interpolational polynomial is constructed for $\Phi(x)$, which, in the special case of odd N ($N = 2i + 1$) considered here, has the form (the fact that $\Phi(x)$ is an odd function is taken into account)

$$\tilde{\Phi}(\theta) \approx \frac{4}{2i+1}\sum_{n=1}^{i}\tilde{\Phi}(\theta_n)\sum_{m=1}^{i}\cos(2m-1)\theta_n\cos(2m-1)\theta$$
$$0 \le \theta \le \pi, \quad \tilde{\Phi}(\theta) = \Phi(\cos\theta) \quad (2.3)$$

Substituting expression (2.3) into Eq. (1.6) and using the collocation method we arrive at the following system of algebraic equations for determining the quantities $\tilde{\Phi}(\theta_n)(n = 1, 2, \ldots, i)$:

$$\sum_{n=1}^{i}\tilde{\Phi}(\theta_n)\left(\Psi_i^-(\theta_n,\theta_k)+\frac{1}{2}\left[F\left(\frac{\cos\theta_n-\cos\theta_k}{\varepsilon}\right)-F\left(\frac{\cos\theta_n+\cos\theta_k}{\varepsilon}\right)\right]\right) =$$
$$= \left(i+\frac{1}{2}\right)\tilde{g}(\theta_k), \quad k = 1, \ldots, i$$
$$\Psi_i^-(\theta_n,\theta_k) = \sum_{m=1}^{i}\frac{\cos(2m-1)\theta_n\cos(2m-1)\theta_k}{m-1/2}, \quad \tilde{g}(\theta_k) = -\frac{\varepsilon}{1-\nu^2}\cos\theta_k \quad (2.4)$$



Numerical results for $\nu = 0.3$ and different values of ε for the contact shear stress are shown in the figure ($\varphi(\theta) = -\varphi(\pi - \theta)$).

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